APPLICATION OF SCALE INVARIANCE PROPERTIES OF RAINFALL FOR ESTIMATING THE INTENSITY-DURATION-FREQUENCY RELATIONSHIPS AT UBERABA, IN SOUTH-CENTRAL BRAZIL

APLICACIÓN DE LAS PROPIEDADES DE INVARIANZA DE ESCALA DE LLUVIAS PARA LA ESTIMACIÓN DE LA RELACIÓN INTENSIDAD-DURACIÓN-FRECUENCIA EN UBERABA, EN EL CENTRO-SUR DE BRASIL

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Abstract

In this paper, the properties of scale invariance of rainfall, in time domain, are investigated and applied in order to estimate the intensity-duration-frequency (IDF) relationships for the town of Uberaba, located in the Brazilian State of Minas Gerais, where only records of daily rainfall depths are available. The assumption of simple scaling (scale) implies in direct relationships between the moments of different orders and the rainfall durations, which can be used to derive IDF relationships from larger durations. As short-duration rainfall records are not available at the Uberaba gauging station, the simple scale invariance properties are verified for sites with sub-daily records and then transferred to the location of interest. At all recording gauging sites investigated in here, the plots of moments of order q versus durations revealed an inflection point around the duration of 1 hour, which indicates a distinction between the prevalent types of precipitation according to their respective durations. This common behavior was kept in transferring the short-duration rainfall information to the location of Uberaba. The resulting IDF relationships were then compared to IDF estimates at locations nearby and the results are discussed. The paper's conclusions also discuss the attributes and limitations of the estimation method.
Keywords: Heavy rainfalls, Scale invariance, IDF relationships.

Resumen

En este artículo, se estudian y aplican las propiedades de invarianza de escala de lluvias en el dominio del tiempo, con el propósito de estimar la relación intensidad-duración-frecuencia (IDF) para la ciudad de Uberaba, ubicada en el Estado brasileño de Minas Gerais, donde únicamente se dispone de datos de densidad diaria de lluvia. La premisa de invarianza simple de escala involucra una vinculación directa entre los momentos de distintos órdenes y la duración de lluvia, lo cual puede ser empleado para deducir las relaciones IDF a partir de las duraciones más largas. Como los datos de densidad sub-diaria de lluvia no se hallan disponibles en la estación pluviométrica de Uberaba, las propiedades de invarianza simple de escala son previamente verificadas con datos sub-diarios obtenidos en sitios relativamente cercanos, y después transpuestas para el sitio de interés. En todas las estaciones con registros de alturas sub-diarias de lluvia aquí estudiadas, los gráficos de los momentos de orden q versus las duraciones, revelan un punto de inflexión alrededor de la duración de 1 hora, el cual sugiere un comportamiento distinto entre tipos prevalentes de precipitación en conformidad a sus respectivas duraciones. Este comportamiento común fue mantenido en la transferencia de la información de las lluvias de corta duración para la localidad de Uberaba. A continuación, las relaciones IDF así alcanzadas fueran confrontadas con estimaciones IDF de sitios cercanos y los resultados son aqui analizados. Finalmente, las conclusiones de este artículo presentan una discusión de los atributos y limitaciones del método de estimación propuesto.
Palabras clave: Lluvias intensas, Invarianza de escala, Relación IDF.

1 – INTRODUCTION

The intensity-duration-frequency (IDF) relationship of heavy rainfalls is certainly among the hydrologic tools most utilized by engineers to design storm sewers, culverts, retention/detention basins, and other structures of storm water management systems. An at-site IDF relationship is a statistical summary of rainfall events, estimated on the basis of records of intensities abstracted from rainfall depths of sub-daily durations, observed at a particular recording rainfall gauging station. At some particular site of interest, there might be one or more recording rainfall gauging stations operating for a time period sufficiently long to yield a reliable estimate of the at-site IDF relationship. In other locations, however, these recording stations may either not exist or have too few records to allow a reliable estimation of IDF relationships.

Because daily precipitation data are by far the most accessible and abundant source of rainfall information, it is appealing to develop methods to derive the IDF characteristics of short-duration events from daily rainfall statistics. The early attempts to derive short-duration rainfall intensities from daily data made use of empirical proportionality factors, which are supposed to be valid at a specific location or over a par-

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ticular geographic region (see for instance Bell, 1969; DAEE/CETESB, 1980). More recently, research has focused on the mathematical representation of rainfall fields both in time and space, including the development of scaling invariance models to derive short-duration rainfall intensity-frequency relations from daily data (see for example Burlando and Rosso, 1996; Menabde et al., 1999; De Michele et al., 2002).

Scaling invariance occurs when the connections among the statistical descriptors of a given phenomenon at different scales are constant and defined by a scale factor. The statistical descriptors can be scaled either by a single factor (simple scaling) or by a more complex function of the scale (multiscaling). As rainfall is concerned, scaling its statistical descriptors in time/space is related to the study of its fractal properties or, in other terms, the way in which rainfall organizes itself in self-affine cell clusters in time/space (Bara et al., 2009). For more theoretical details on scale invariance properties and rainfall IDF relationships, the reader is referred to Burlando and Rosso (1996) and Menabde et al. (1999).

This paper aims (i) to present a brief introduction of scale invariance as applied to rainfall events, (ii) in order to derive the IDF relationship from daily data for the town of Uberaba, located in south-central Brazil, and (iii) finally provide some discussions of the results and some concluding remarks. The paper is organized in six additional sections. The next section provides a description of the available data at Uberaba and outlines the sequential steps to be followed in this case study. Section 3 provides the necessary theoretical background on the simple scale invariance as applied to IDF estimations. The next two sections describe the application of simple scaling properties to estimating the IDF relationship from the daily rainfall data available at Uberaba. Analysis of the results and the main conclusions are provided in the last section.

2 – THE AVAILABLE DATA

The data available for the study described herein are the daily rainfall depths observed at the gauging station located in Uberaba, a town of 296,000 inhabitants located in the Brazilian state of Minas Gerais, in south-central Brazil, at coordinates 19°44'52'' south and 47°55'55", as illustrated in Figure 1. The rainfall gauging station is operated by the Brazilian National Institute of Meteorology (INMET) under the code 83577. The period of available data spans from 1914 to 2012, with 19 incomplete years containing one or more missing data during the rainy season (from October to March). Figure 2 depicts a chart of the 80 annual maximum daily rainfall depths, according to their chronology in calendar years. In order to preserve the essential statistical features of at-site maximum daily rainfall, missing data for incomplete years were not filled in by data available at nearby stations.

In addition to the annual maximum rainfall depths, depicted in Figure 2, two other sources of information were available for this study. The first one refers to the IDF equations available for two locations relatively close to Uberaba: the one estimated for Barretos, at coordinates 20°33'26'' south and 48°34'04" west, and the other for the location of Catalão, at coordinates 18°09'57" south and 47°56'47" west. These locations are also indicated by arrows in Figure 1. The second source refers to the IDF equation estimated to the town of Uberaba by Freitas et al. (2001), through interpolation of regional data observed at some recording gauging stations within the entire state of Minas Gerais, the boundaries of which extend over a total surface area of 586,528 square kilometers.

As at-site sub-daily rainfall data were not available, estimation of the IDF relationship for Uberaba from the available records would have necessarily to rely on some strategy to disaggregate daily data into sub-daily quantities. Among the common strategies are (a) the use of empirical proportionality factors and (b)

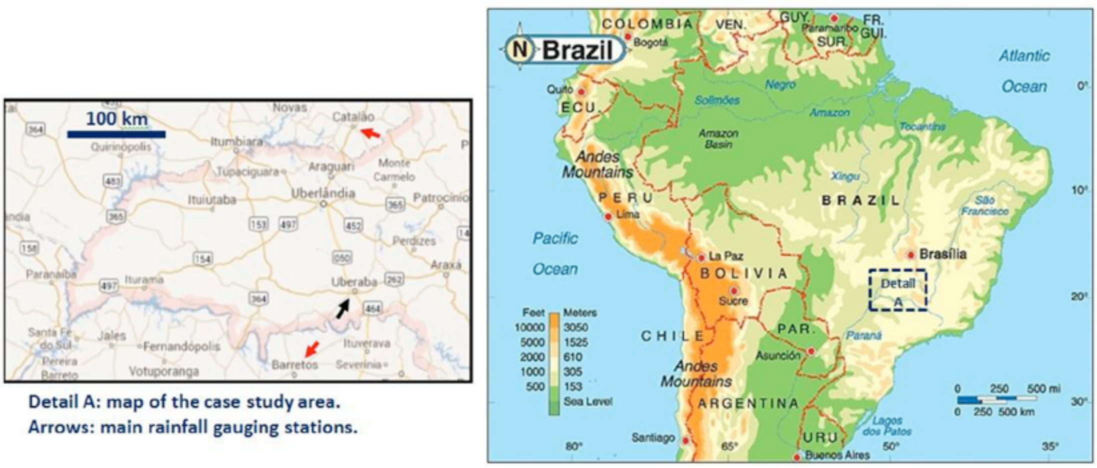

Figure 1 – Location of Uberaba, in south-central Brazil. (http://maps.google.com.br)

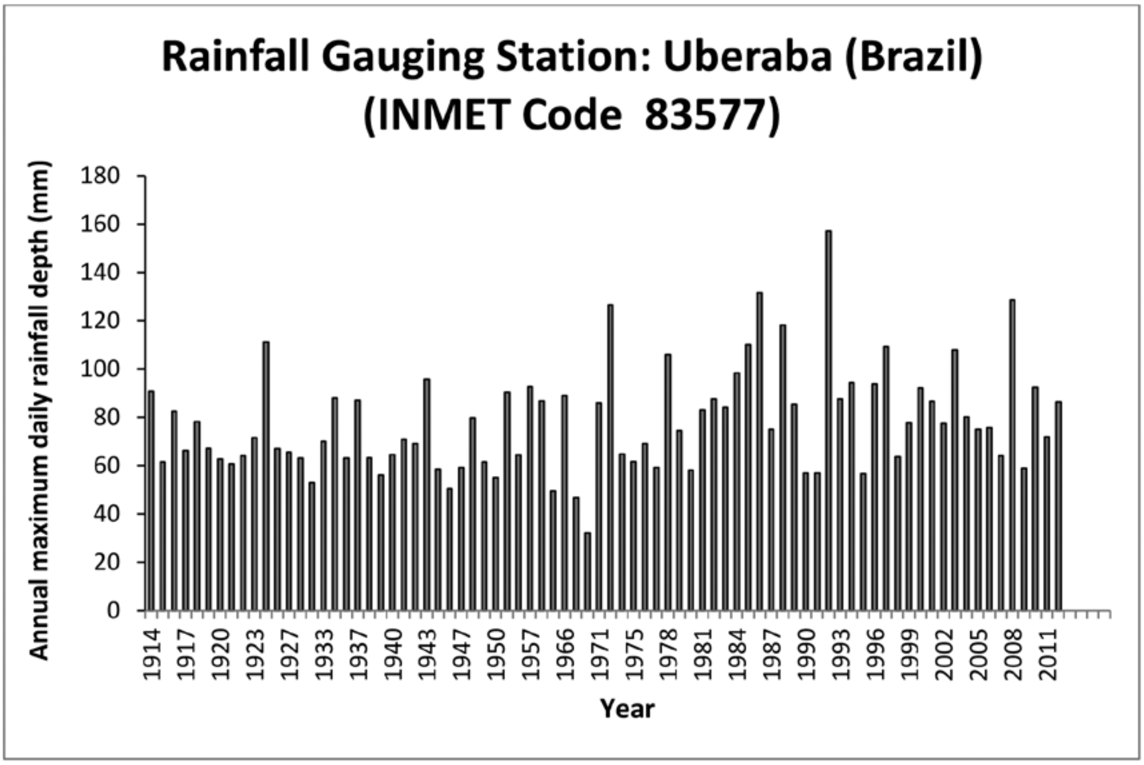

Figure 2 – Annual maximum daily rainfall depths, in millimeters per day, observed at the gauging station of Uberaba (INMET code 83577)

the use of scale invariance properties between daily and sub-daily intensities. In the present study, strategy (b) has been selected for the following reasons:

- In the last two decades, the study of scale invariance properties and their use to IDF estimation have been the focus of many theoretical and applied works, such as Gupta and Waymire (1990), Burlando and Rosso (1996), Menabde et al. (1999), Naghettini (2000), Gerold and Watkins (2005), Minh Nhat et al. (2008) and Bara et al. (2009), thus providing a consistent method for dealing with temporal statistical downscaling of hydrometeorological variables;
- Application of scale invariance properties to IDF estimation has been performed for a number of locations in the Brazilian state of Minas Gerais, providing good results with relatively simple implementation procedures (Naghettini, 2000);
- The so-called proportionality factors (DAEE/CETESB, 1980), calculated as empirical ratios between sub-daily and daily rainfall data averaged among a number of sites, are in fact generic quantities that do not account for the local features affecting rainfall at a given location and for the statistical dependence that may exist between these ratios and the return periods.

The next section presents the theoretical foundations of simple scale invariance, as employed in estimating IDF relationships of short duration point rainfall.

3 – THEORETICAL BACKGROUND

According to Koutsoyannis et al. (1998), for a given return period, IDF relationships are particular cases of the following general formula

$$i = \frac{w}{\left(d^{\nu} + \theta\right)^{\eta}} \tag{1}$$

where i denotes the rainfall intensity of duration d, and w, ν, θ, and η are non-negative coefficients. Koutsoyannis et al. (1998) also proposed a numerical exercise, in which they show that the errors resulting from imposing ν=1 in equation (1) are much smaller than the typical parameter and quantile estimation errors from limited size samples of rainfall data. Hence, considering $\nu\neq1$ as a model over-parameterization, Koutsoyannis et al. (1998) prescribe that, for a given return period, the general expression for IDF relationships should be written as

$$i = \frac{w}{\left(d + \theta\right)^{\eta}} \tag{2}$$

Rigorously, the coefficients w, θ, and η depend on the return period. However, because the IDF curves for different return periods cannot intersect each other, this dependence cannot be arbitrary. Actually, this restriction imposes bounds for the range of variation of parameters w, θ, and η. If $\{w_1, \theta_1$ e $\eta_1\}$ and $\{w_2, \theta_2$ e $\eta_2\}$ denote the parameter sets for return periods T_1 and T_2, respectively, with $T_2<T_1$, Koutsoyannis et al. (1998) suggest the following restrictions on the range of variation of parameters

$$\begin{aligned} &\theta_1 = \theta_2 = \theta \geq 0 \\ &0 < \eta_1 = \eta_2 = \eta < 1 \\ &w_1 > w_2 > 0 \end{aligned} \tag{3}$$

In this set of restrictions, it is worth to note the only parameter that can consistently increase with increasing return periods is w, which results in substantial simplification of equation (2). In fact, these arguments justify the formulation of the following general model for IDF relationships

$$i_{d,T} = \frac{a(T)}{b(d)} \tag{4}$$

which exhibits the advantage of expressing separable dependence relations between i and T, and between i and d. In equation (4), $b(d)=(d+\theta)^{\eta}$ with $\theta>0$ and $0<\eta<1$, whereas $a(T)$ is completely defined by the probability distribution function of the maximum rainfall intensities. The analytical form of equation (4) is consistent with most IDF equations estimated for many locations in Brazil and elsewhere [see, for instance, Wilken (1978), Pinheiro and Naghettini (1998), Chen (1983) and Raiford et al. (2007)].

Suppose that I_d denote the annual maximum rainfall intensity of duration d, defined as the ratio between the annual maximum total depth, abstracted for the time duration d, and the duration d itself. The random variable I_d has a cumulative probability function $F_d(i)$, which is given by

$$\Pr(I_d \leq i) = F_d(i) = 1 - \frac{1}{T_{(i)}} \tag{5}$$

where T represents the return period, in years, associated with the event.

Rainfall fields may show the property of 'simple scaling in the strict sense', which is formally defined by Menabde et al. (1999) by the expression

$$I_d \overset{dist}{=} \left(\frac{d}{D}\right)^{-\gamma} I_D \tag{6}$$

where the sign of equality refers to identical probability distributions in both sides of the equation, D denotes a duration $D>d$, usually taken as 24 hours, and γ is the scale factor which is supposed constant. As opposed to the simple scaling hypothesis, there is the general case of 'multiscaling', in which the factor $(d/D)^{-\gamma}$ is regarded as a random variable dependent upon the ratio (d/D). The simple scaling hypothesis, as defined by equation (6), can be empirically verified and, if accepted as true, be employed to construct simple disaggregation models of practical use.

Equation (6) may be rewritten in terms of the moments of order q about the origin, denoted by, $\langle I_d^q \rangle$ thus resulting the expression

$$\langle I_d^q \rangle = \left(\frac{d}{D}\right)^{-\gamma q} \langle I_D^q \rangle \tag{7}$$

or

$$d^{\gamma q} \langle I_d^q \rangle = D^{\gamma q} \langle I_D^q \rangle \tag{8}$$

According to Menabde et al. (1999), the only functional form of $\langle I_d^q \rangle$ capable of satisfying equation (8) is

$$\langle I_d^q \rangle = G(q)\, d^{-\gamma q} \tag{9}$$

where $G(q)$ is a function of q. This expression reflects the property of 'simple scaling in the wide sense', meaning that equation (9) is implied by equation (6) but not vice-versa. In the case of 'multiscaling', the exponent of d, as in equation (9), would have to be replaced by a non-linear function $K(q)$.

From equation (6), it follows that

$$F_d(i) = F_D\left[\left(\frac{d}{D}\right)^{\gamma} i\right] \tag{10}$$

For many parametric forms, equation (10) may be expressed in terms of a standard variate, as given by

$$F_d(i) = F\left(\frac{i-\mu_d}{\sigma_d}\right) \tag{11}$$

where $F(.)$ is a function independent of d. If that is the case, it follows from equation (10) that

$$\mu_d = \left(\frac{d}{D}\right)^{-\gamma} \mu_D \tag{12}$$

and

$$\sigma_d = \left(\frac{d}{D}\right)^{-\gamma} \sigma_D \tag{13}$$

By substituting expressions (11), (12), and (13) into equation (5) and inverting it with respect to i, one obtains

$$i_{d,T} = \frac{\mu_D D^{\gamma} - \sigma_D D^{\gamma} F^{-1}(1-1/T)}{d^{\gamma}} \tag{14}$$

By equaling equation (14) to the general model for IDF relationships, as in equation (4), it is easy to verify that

$$\gamma = \eta \tag{15}$$

$$\theta = 0 \tag{16}$$

$$b(d) = d^{\eta} \tag{17}$$

and

$$a(T) = \mu + \sigma F^{-1}(1 - 1/T) \tag{18}$$

where $\mu = \mu_D D^{\eta}$ and $\sigma = \sigma_D D^{\eta}$ are constants. It is worthwhile to note that the simple scaling hypothesis implies the equality between the scale factor γ and the exponent η, as in the expression relating rainfall intensities and durations.

The simple scaling property, as formalized by equation (9), can be empirically verified by replacing the population moments by the corresponding sample moments about the origin. On the other hand, in order to check the validity of equation (10), one needs to specify a probability distribution for the annual maximum rainfall intensities. In this context, two of the most frequently used probability distributions, namely the Generalized Extreme Value (GEV) and the EV1 or Gumbel parametric models, are examples of functional forms that are compatible with expression (11) and appropriate for the empirical verification of equation (10).

4. APPLICATION OF SIMPLE SCALING PROPERTIES

The adequacy of the simple scaling model to IDF estimation, as prescribed by equation (14), can be verified through the use of sub-daily data from recording gauging stations. However, because sub-daily rainfall intensity data were not available at the Uberaba gauging station, the studies concerning the scale invariance properties were performed for four recording gauging stations located in different regions, within the state of Minas Gerais, in order to make possible transferring the rainfall information to the site of interest. These stations are: Vespasiano (code ANA 01943009), in the metropolitan region of the state capital Belo Horizonte; Papagaios (code ANA 01944049); Lagoa do Gouvea (code ANA 01845004); and Entre Rios de Minas (code ANA 02044007), all of them located in the upper São Francisco river basin, which borders, on the northeast side, the Grande river watershed, where Uberaba is situated.

Taking into account (i) that the main simple scaling properties, namely, the exponent η, as in equation (15), and the durations for which these properties prevail, did not show much variation among the stations (Naghettini, 2000); (ii) that the gauging station of Vespasiano, as opposed to the other three sites, is located within a micro-region with less pronounced relief, which roughly approximates the orographic features found in Uberaba; and (iii) that are available 18 years of rainfall data for the durations 10 min, 15 min, 30 min, 45 min, 1, 2, 3, 4, 8, 14 e 24 hours at the Vespasiano gauging station, the decision was to apply the scale invariance model to this station and then combine the results with the daily rainfall data available at Uberaba in order to derive the local IDF relationship. The application of scale invariance model to Vespasiano rainfall data is the object of the paragraphs to follow.

The verification of the simple scaling model can be done by using the sample moments in equation (9), with γ=η. Figure 3 illustrates the association of $\langle I_d^q \rangle$ with *d*, both in logarithmic space, for moment orders *q*=1, 2, and 3. Note that for all orders *q*, there are well defined scale relationships for durations from 1 to 24 hours; the durations smaller than 1 hour exhibit the same property, but with a distinct slope. In fact, the regressions between moments and durations, in logarithmic coordinates, conform better to linearity when they are separated in a first subset for durations from 24 hours down to 1 hour, and a second subset for sub-hourly durations. Figures 4 and 5 depict the scatter plots and the regression results for subsets of durations from 24 to 1 hour and from 1 hour to 10 minutes, respectively. In both cases, it is evident the linear fit conforms better for two distinct subsets than for the entire set of durations.

By representing the exponent of *d* in equation (9) by $K(q) = -\eta q$ and using the coefficients of the regressions between log [$\langle I_d^q \rangle$] and log (*d*), valid for the two subsets of durations, it is clear from Figures 6 and 7, the relations between $K(q)$ and *q* are very closely linear. The linearity of these relations is an argument to confirm the hypothesis of simple scale invariance in a wide sense, as formalized by equation (9), for the two subsets of durations. The angular coefficient of the linear regression between $K(q)$ and *q* gives the following estimates of the scale factor: $\hat{\eta}$ = 0,7398, for durations from 1 hour to 24 hours, and $\hat{\eta}$ = 0,5681, for sub-hourly durations, with an intersection point exactly at *d*=1 hour. The derivation of IDF estimates from 24-hour rainfall can be accomplished by using equation (14), with $\gamma = \hat{\eta} = 0{,}7398$ and with the estimates of μ_D and σ_D, for *D*=24 hours. For sub-hourly durations, the procedure is identical, this time with γ=$\hat{\eta}$=0,5681 and with the estimates μ_D and σ_D for *D*=1 hour. It is worth to remark that μ_D and σ_D represent respectively the location and scale parameters of the probability distribution fitted to the rainfall intensities of duration *D.* Taking as an example the Vespasiano gauging station, where the Gumbel distributions fits well the rainfall intensities of 24 hours with parameters μ_{24}=3,17mm/h and σ_{24}=1,1333 mm/h, it is easy to see, by employing the Gumbel inverse function in

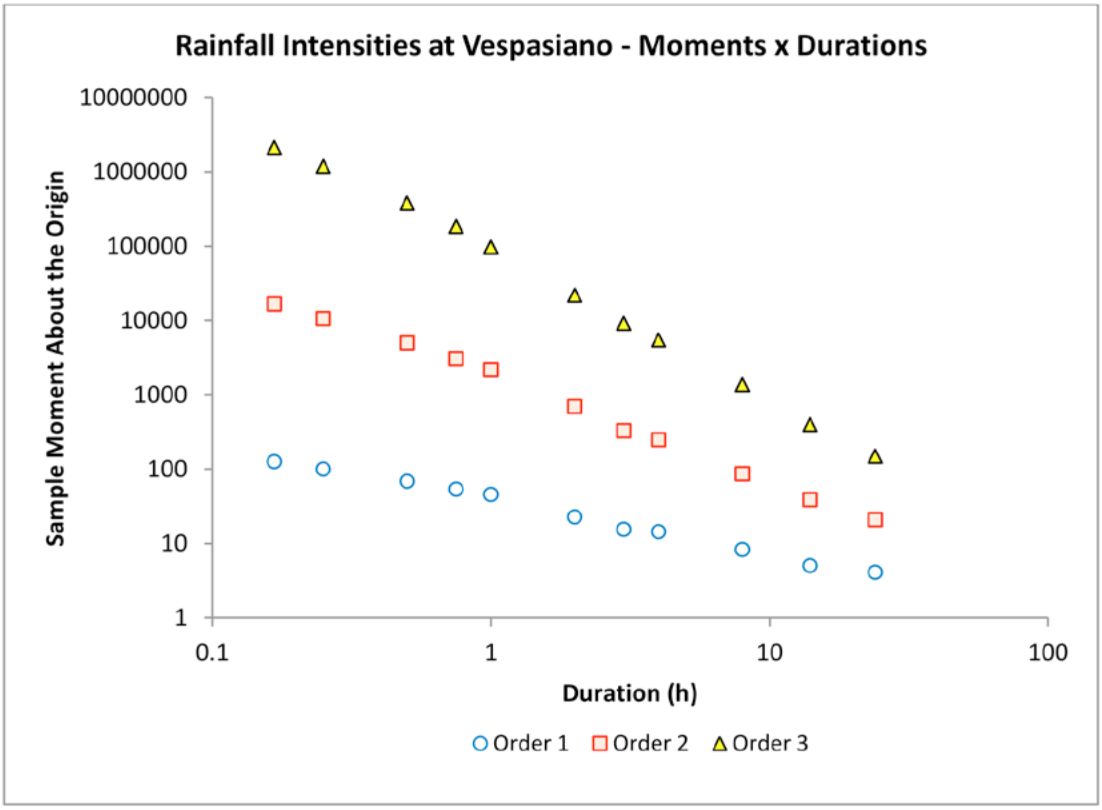

Figure 3 – Sample moments of order q versus durations, for sub-daily annual maximum rainfall intensities recorded at Vespasiano. (10 min ≤ d ≤ 24 h)

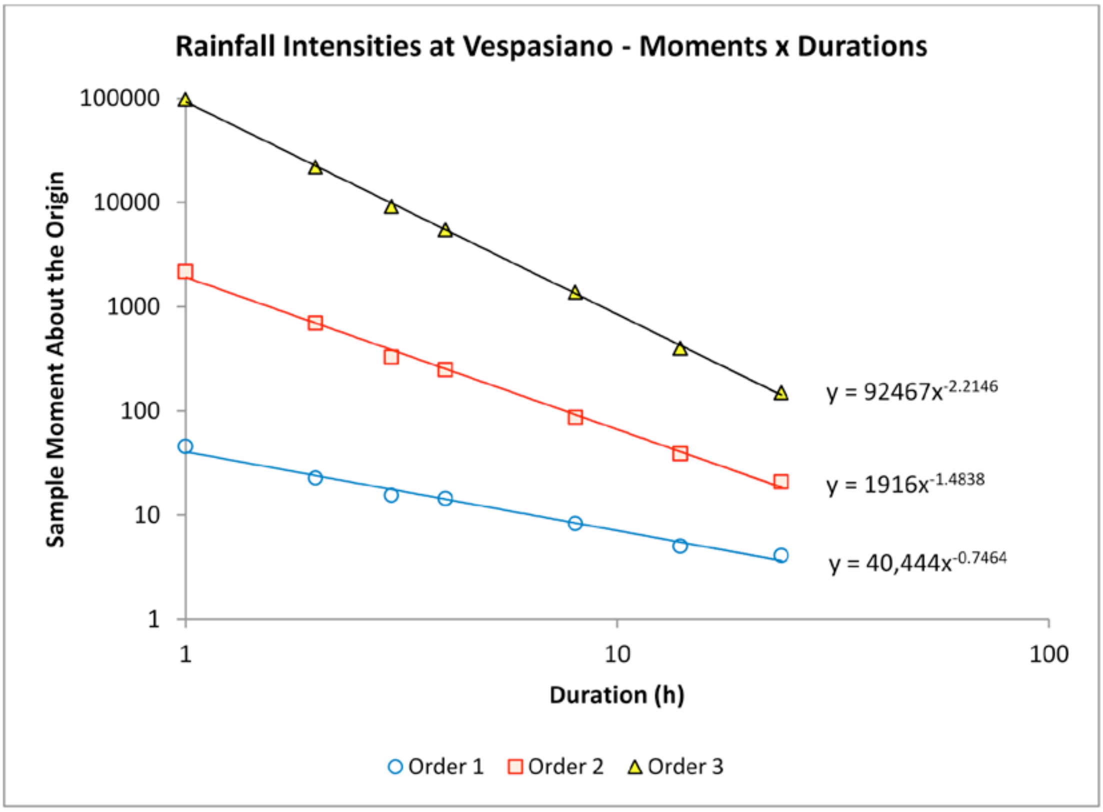

Figure 4 – Sample moments of order q versus durations, for sub-daily annual maximum rainfall intensities recorded at Vespasiano. (1 h ≤ d ≤ 24 h)

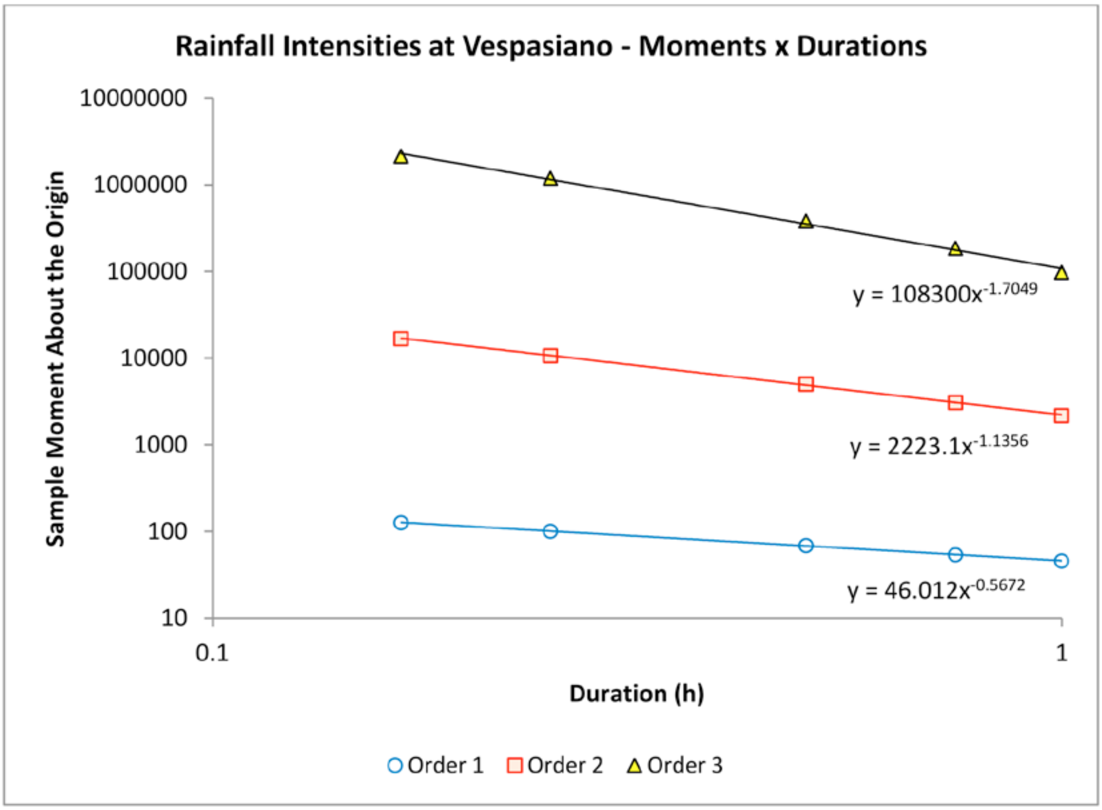

Figure 5 – Sample moments of order q versus durations, for sub-daily annual maximum rainfall intensities recorded at Vespasiano. (10 min ≤ d ≤ 1 h)

equation (14), that the corresponding IDF estimates for durations in the range of 1 h to 24 h, should be given by

$$i_{d,T} = \frac{28.4 - 10.16\,ln\left[-\,ln\left(1 - 1/T\right)\right]}{d^{0.7398}} \qquad (19)$$

This is the essence of applying the simple scale invariance model to IDF estimation. On the basis of the arguments provided, the strategy to derive the IDF relationship for Uberaba comprehends the following steps:

- From the relief characteristics of Vespasiano and from the small spatial variability of the scale factors within the region under study [for instance, the scale factors, as calculated by Naghettini (2000), for durations larger than 1 hour for the sub-daily data observed at Papagaios (01944049), at Lagoa do Gouvea (01845004), and at Entre Rios de Minas (02044007) were 0.75, 0.77, and 0.81, respectively], it seems plausible to admit as valid, also for Uberaba, the following estimates for the scale factor: ή=0,7398, for durations from 1 hour to 24 hours, and ή=0,5681, for sub-hourly durations;
- From the series of annual maximum daily rainfall intensities, observed at the gauging station of Uberaba (INMET 83577), the second step is to build the series of annual maximum 24-hour rainfall intensities, by multiplying the first one by the factor 1.14, as recommended by DAEE/CETESB (1980), as the empirical ratio between 24-hour and daily rainfall depths averaged among a large number of sites;
- The next step is to fit a parametric probability model, which should be compatible with the inherent assumptions of simple scaling invariance, such as Gumbel, Log-Normal, or GEV (Generalized Extreme Value) distributions, to the annual maximum 24-hour rainfall intensities as built in the previous step; and
- Finally, to derive the IDF relationship for the location of Uberaba, by using equation (14) first for durations from 24-hour down to 1-hour, with the scale factor ή=0,7398, and next, for durations from 1 hour to 10 minutes, with the scale factor ή=0,568, both with an intersection point at *d*=1 hour.

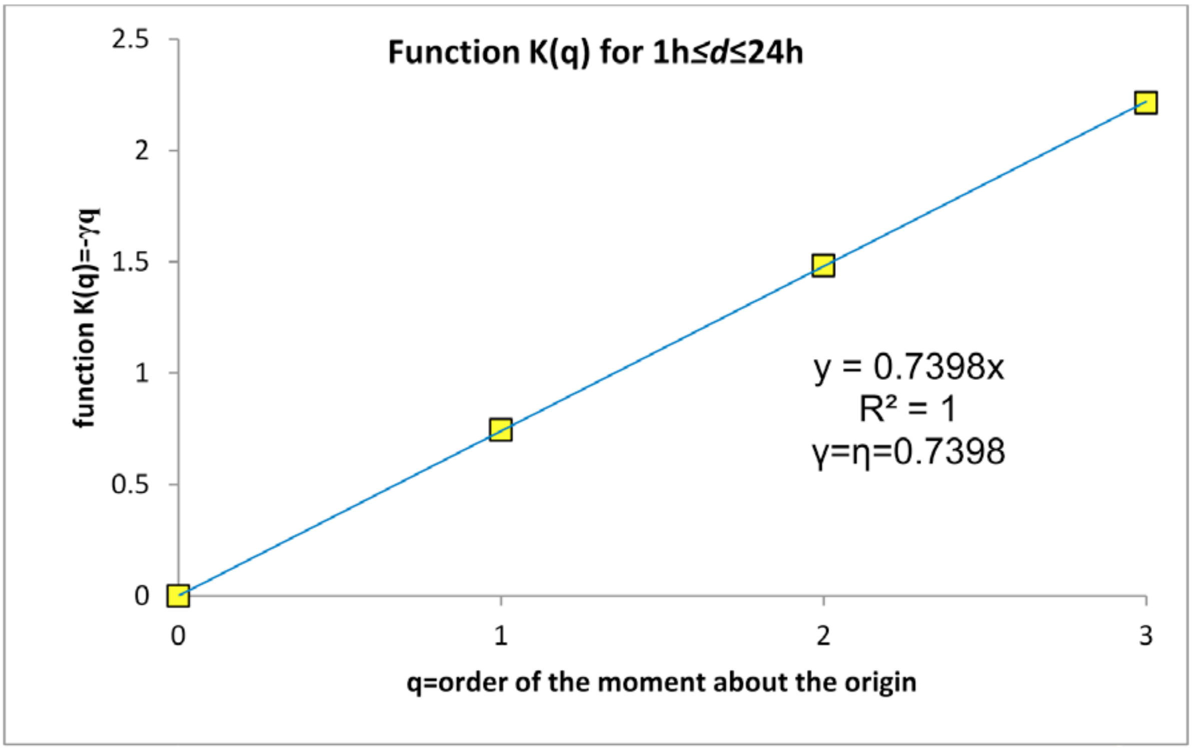

Figure 6 – Function K(q)=-γq for the sample moments of order q of rainfall intensities of durations from 1 h to 24 h, at the Vespasiano gauging station.

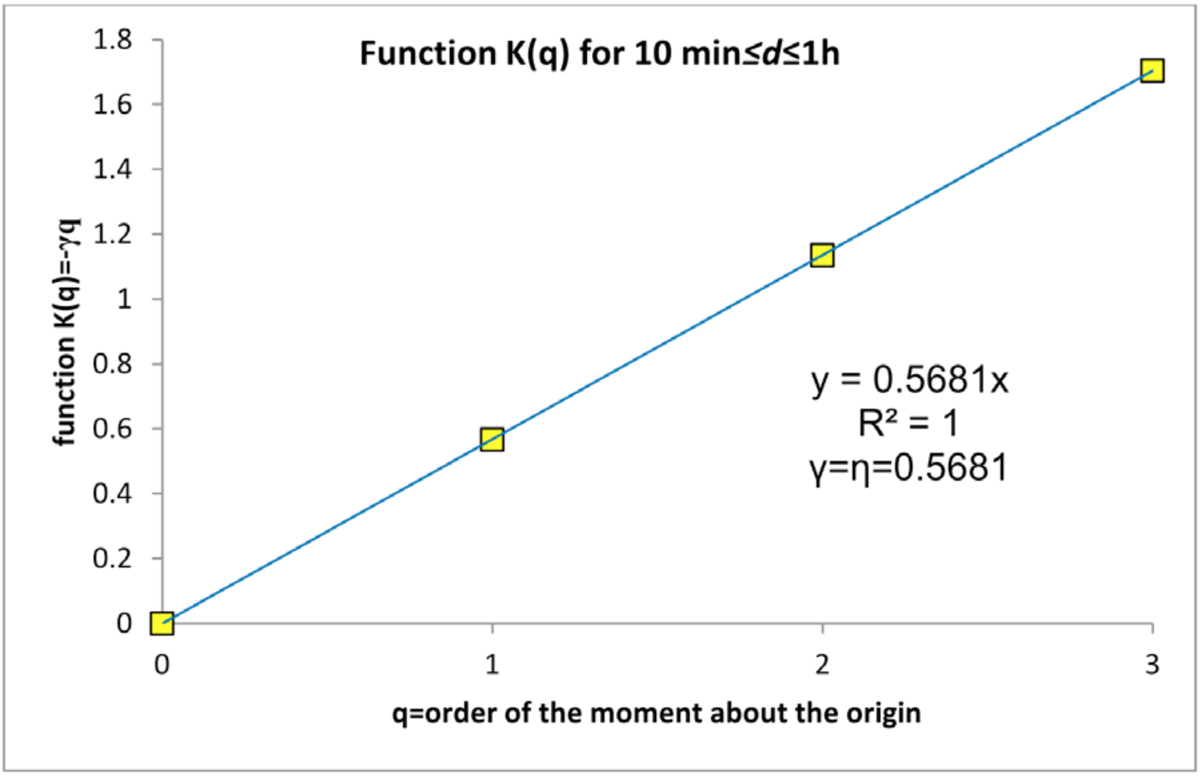

Figure 7 – Function K(q)=-γq for the sample moments of order q of rainfall intensities of durations from 10 min to 1 h, at the Vespasiano gauging station.

5 – FREQUENCY ANALYSIS OF ANNUAL MAXIMUM 24-HOUR RAINFALL INTENSITIES

Table 1 presents some descriptive statistics for the 80 values of annual maximum 24-hour rainfall intensity (mm/h), as calculated by the multiplication of the daily series, available at the gauging station of Uberaba from 1914 to 2012, by the factor 1.14. Figure 8 depicts the histogram of the available sample, with 13 class intervals.

Table 1 – Descriptive statistics for the sample of annual maximum 24-hour rainfall intensities at Uberaba. Except for dimensionless quantities, units are mm/h.

N	**80**	**3rd Quartile**	**4.212**
Mean	3.703	Maximum	7.458
Std. Deviation	1.029	Range	5.928
Coeff. Of Variation	0.278	Inter-quartile Range	1.233
Minimum	1.529	Mode	4.161
1st Quartile	2.979	Skewness	1.046
Median	3.548	Kurtosis	1.651

The sample of annual maximum 24-hour rainfall intensities has been submitted to a preliminary analysis for detecting possible serial dependence, heterogeneity, and outliers, respectively, through the statistical significance tests of Wald-Wolfowitz, Mann-Whitney and Grubbs-Beck. The null hypothesis of statistical independence cannot be rejected at 5% significance level (Wald-Wolfowitz test statistic=0.0012 with p-value=0.4995). The hypothesis of homogeneity also cannot be rejected at α=5% (Mann-Whitney test statistic=-1.887 with p-value=0.0296). However, as the presence of outliers is concerned, the Grubbs-Beck test, at α=5%, identified the 1969 rainfall intensity of 1.53 mm/h as a low outlier; no high outlier was identified. Since a low outlier can potentially affect the upper-tail estimation, which is the main focus of this work, the 1969 value has been withdrawn from the sample.

In the sequence, 2-parameter probability models, such as Exponential, Gumbel, and Log-Normal, as well as 3-parameter models, such as GEV, Pearson III, and Log-Pearson III, have been fitted to the sample by using the method of moments. All fitted models have passed the Kolmogorov-Smirnov (KS) and Chi-squared (χ^2) goodness-of-fit tests at the 5% significance level. On an exponential probability paper, the best fits were achieved by the GEV and the Gumbel probability distributions (Figure 8). The

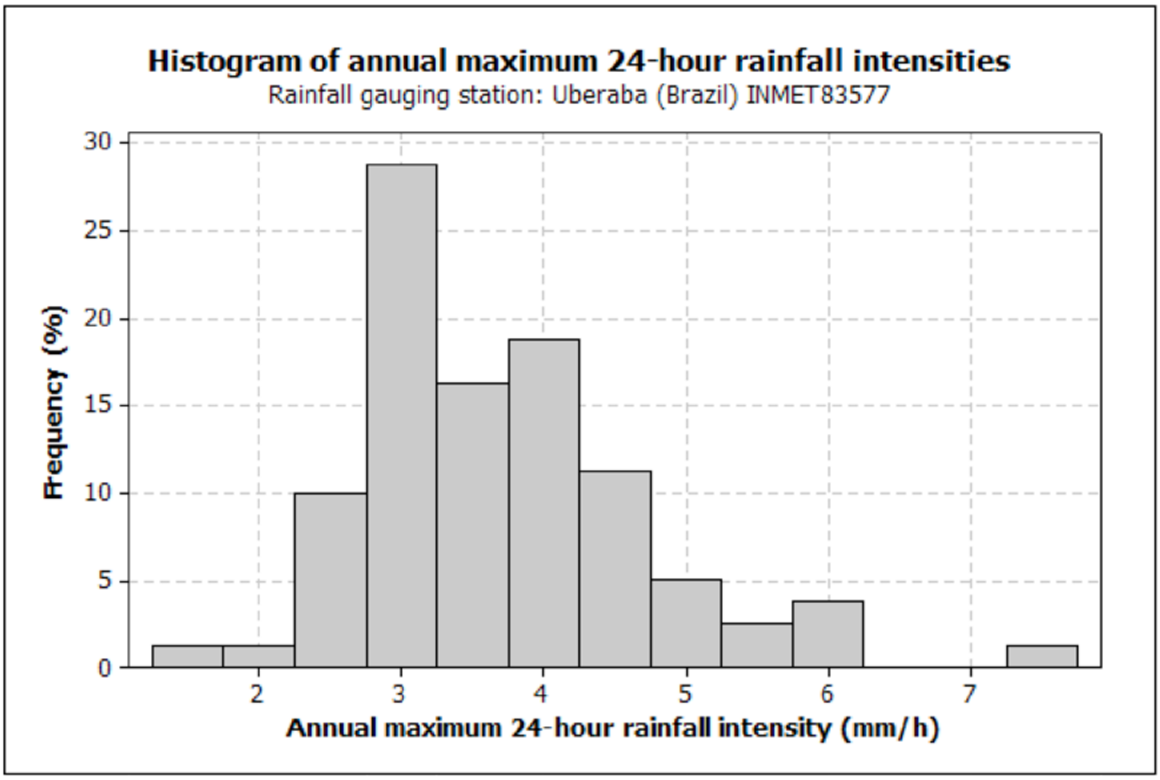

Figure 8 – Histogram of annual maximum 24-hour rainfall intensities at Uberaba.

goodness-of-fit statistics for the GEV model were KS statistic=0.0798, with p-value=0.6781; χ^2=10.7314 with p-value=0.903 and 5 degrees of freedom, whereas for the Gumbel distribution, they were KS statistic=0.0794, with p-value=0.6831; χ^2=10.7417 with p-value=0.950 and 6 degrees of freedom. Figure 9 shows a chart with the empirical frequency curve, and the fitted GEV and Gumbel models, along with their respective 95% confidence intervals.

The Gumbel distribution, as fitted by the method of moments, with parameter estimates $\hat{a} = 0{,}784$ and $\hat{a} = 3{,}278$, has been selected as the best-fit model. The selection of this model was based on the following arguments: (i) the Gumbel model provides a very good fit to empirical data; (ii) the Gumbel and GEV quantiles are almost identical to each other, for return periods up to 200 years; (iii) the 2-parameter Gumbel model is more parsimonious, in the statistical sense, than the 3-parameter GEV model; (iv) the Gumbel fit has narrower confidence intervals than those yielded by the GEV distribution; and (v) it is consistent with the simple scaling formulation given by equation (14).

6 – ESTIMATING THE IDF RELATIONSHIP FOR UBERABA – PROCEDURES AND RESULTS

The Gumbel distribution has as probability density function the expression

$$f_X(x) = \alpha \exp\left\{-\alpha(x-\beta) - \exp\left[-\alpha(x-\beta)\right]\right\}$$

$$-\infty < x < \infty, -\infty < \beta < \infty \text{ e } \alpha > 0 \quad (20)$$

and its cumulative probability function is given by

$$F_X(x) = exp\left[-exp\left(-\frac{x-\beta}{\alpha}\right)\right] \quad (21)$$

where α and β are scale and location parameters, equivalent, in equation (14), to σ_D and μ_D, respectively. Replacing F^{-1} in equation (14) by the Gumbel inverse function, with the usual parameter notation, in terms of the return period *T*, one can write

$$i_{d,T} = \frac{\beta_D D^\eta - \alpha_D D^\eta \, ln[-ln(1-1/T)]}{d^\eta} \quad (22)$$

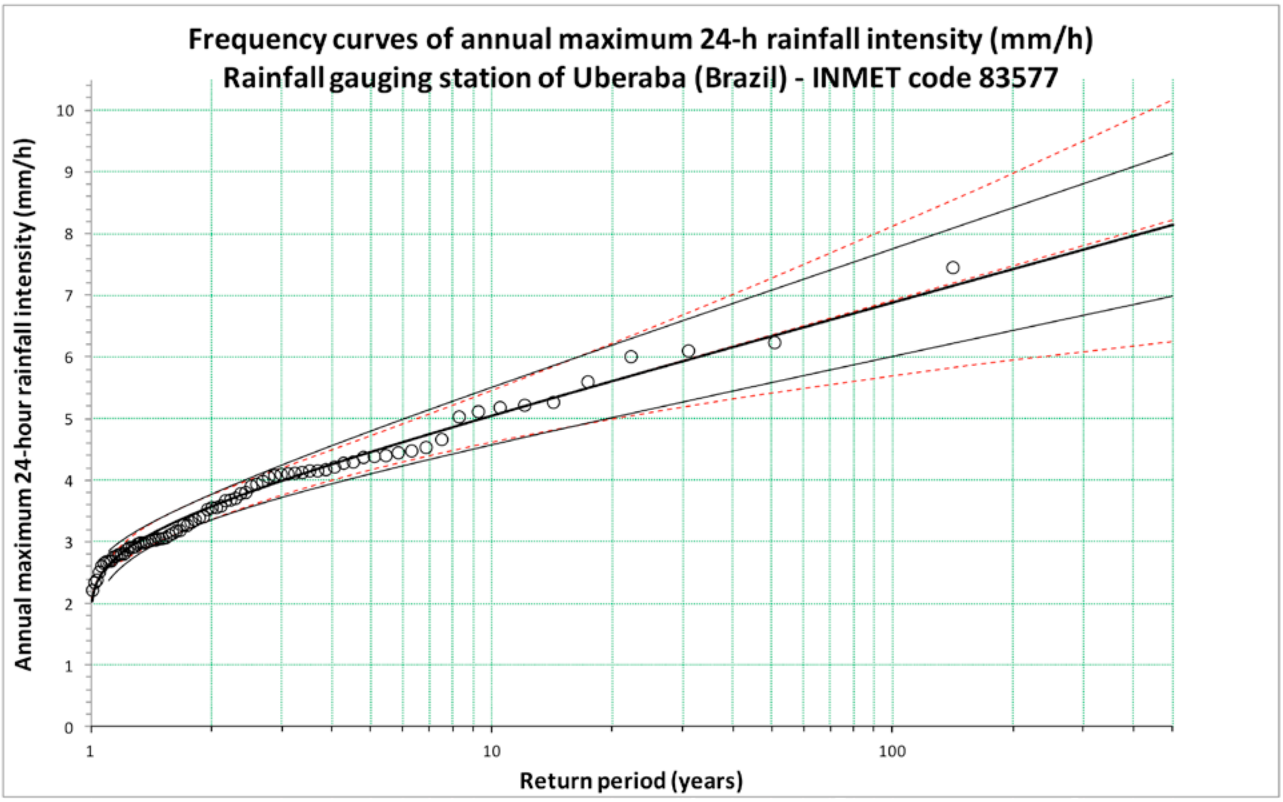

Figure 9 – Gumbel and GEV fits to the sample of annual maximum 24-hour rainfall intensities at the gauge of Uberaba. Gumbel quantiles and 95% confidence intervals are marked in solid lines. GEV quantiles and 95% confidence intervals are marked in dashed lines. Gringorten plotting positions were used for empirical frequencies.

Equation (22) is the key expression for deriving the IDF relationship for sub-daily durations. For the case of Uberaba, initially with D=24 h, $\hat{\alpha}$=0,784, $\hat{\beta}$=3,278, $\hat{\eta}$=0,7398, and 1h ≤ d ≤ 24 h, the IDF equation for durations between 24 hours and 1 hour is given by

$$i_{d,T} = \frac{34{,}410 - 8{,}230\ln\left[-\ln\left(1 - 1/T\right)\right]}{d^{0,7398}} \tag{23}$$

where $i_{d,T}$ represents the intensity in mm/h, of a rainfall of duration d (in hours) and return period T (in years). Analogously, this time with D=1 h, $\hat{\alpha}$=8,230, $\hat{\beta}$=34,410, $\hat{\eta}$=0,5681, and 0,1667 h ≤ d ≤ 1 h, the IDF equation for sub-hourly durations equal or larger than 10 minutes (0,1667 h) is given by

$$i_{d,T} = \frac{34{,}410 - 8{,}230\ln\left[-\ln\left(1 - 1/T\right)\right]}{d^{0,5681}} \tag{24}$$

where $i_{d,T}$ represents the intensity in mm/h, of a rainfall of duration d (in hours) and return period T (in years).

The IDF relationships, as given by equations (23) e (24), had their results compared to those estimated by three other procedures:

- Equation $i_{d,T} = (d+20)^a\left[b + c\ln(T-0{,}5)\right]$, where a=-0,849, b=19,18 e c=5,37 for 10 min ≤ d ≤ 60 min, and a=-0,834, b=17,78 and c=4,98 for 60 min ≤ d ≤ 1440 min, as reported by DAEE/CETESB (1980) for the recording gauging station of Barretos, located in the state of São Paulo, with latitude 20°33' south and longitude 48°34' west, code INMET 83625, and intensities expressed in mm/min and duration in minutes;

- Equation $i_{d,T} = 2400T^{0,164}/(d+31{,}194)^{0,867}$ for the location of Uberaba, defined by Freitas et al. (2001), by interpolating rainfall data from several recording gauging stations located in the state of Minas Gerais, with intensities expressed in mm/h and durations in minutes; and Tables of intensity, duration, and frequency for the location of Catalão, in the state of Goiás, with latitude 18°10' south and longitude 47°58' west, defined by Pfafstetter (1957) and reproduced in DAEE/CETESB (1980).

Tables 2, 3, 4, and 5 show the comparison among the results, as obtained by the equations deduced in this paper and by each of the previously mentioned procedures, for return periods T=2, 25, 50, and 100 years, respectively. Figures 9 to 12 illustrate the graphical comparison among the results from Tables 2 to 5. The tables and figures show the results from the IDF relationships described in here are in between those estimated for the locations of Barretos and Catalão, with moderate percent differences. Concerning the results from the equation proposed by Freitas et al. (2001), they are systematically larger than those yielded by equations (23) and (24), particularly for durations larger than 30 minutes, with relative percent errors ranging from 1.0 to 56.2%.

Table 2 – Comparison among the IDF relationships proposed for Uberaba, for *T*=2 years.

1	2	3	4	5	6	7	8
Duration (h)	**Eqs. (23) and (24)(mm/h)**	**Eq. Barretos (mm/h)**	**2/3**	**Freitas et al. (2001) (mm/h)**	**2/5**	**Eq. Catalão (mm/h)**	**2/7**
0.1667	103.56	71.39	1.45	107.03	0.97	115.08	0.90
0.5	55.49	46.27	1.20	75.95	0.73	63.96	0.87
1	37.43	31.04	1.21	53.74	0.70	41.34	0.91
2	22.41	19.27	1.16	34.67	0.65	25.62	0.88
4	13.42	11.50	1.17	20.89	0.64	15.48	0.87
8	8.04	6.66	1.21	12.06	0.67	9.24	0.87
14	5.31	4.24	1.25	7.59	0.70	6.06	0.88
24	3.57	2.73	1.31	4.82	0.74	4.08	0.87

Table 3 – Comparison among the IDF relationships proposed for Uberaba, for *T*=25 years.

1	2	3	4	5	6	7	8
Duration (h)	Eqs. (23) and (24) (mm/h)	Eq. Barretos (mm/h)	2/3	Freitas et al. (2001) (mm/h)	2/5	Eq. Catalão (mm/h)	2/7
0.1667	168.05	121.52	1.38	161.96	1.04	159.06	1.06
0.5	90.04	78.76	1.14	114.92	0.78	93.72	0.96
1	60.73	52.85	1.15	81.32	0.75	63.36	0.96
2	36.37	32.81	1.11	52.46	0.69	40.26	0.90
4	21.78	19.58	1.11	31.61	0.69	24.84	0.88
8	13.04	11.34	1.15	18.24	0.72	14.88	0.88
14	8.62	7.22	1.19	11.49	0.75	9.36	0.92
24	5.79	4.64	1.25	7.30	0.79	6.24	0.93

Table 4 – Comparison among the IDF relationships proposed for Uberaba, for *T*=50 years.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Duration (h)	**Eqs. (23) and (24) (mm/h)**	**Eq. Barretos (mm/h)**	**2/3**	**Freitas et al. (2001) (mm/h)**	**2/5**	**Eq. Catalão (mm/h)**	**2/7**
0.1667	184.07	134.15	1.37	181.46	1.01	172.50	1.07
0.5	98.63	86.94	1.13	128.76	0.77	103.26	0.96
1	66.52	58.34	1.14	91.11	0.73	70.74	0.94
2	39.84	36.22	1.10	58.77	0.68	45.24	0.88
4	23.85	21.61	1.10	35.42	0.67	28.08	0.85
8	14.29	12.53	1.14	20.44	0.70	16.80	0.85
14	9.44	7.97	1.19	12.88	0.74	10.98	0.86
24	6.34	5.13	1.24	8.17	0.78	7.26	0.87

Table 5 – Comparison among the IDF relationships proposed for Uberaba, for *T*=100 years.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Duration (h)	**Eqs. (23) and (24) (mm/h)**	**Eq. Barretos (mm/h)**	**2/3**	**Freitas et al. (2001) (mm/h)**	**2/5**	**Eq. Catalão (mm/h)**	**2/7**
0.1667	199.97	146.68	1.36	203.31	0.98	186.72	1.07
0.5	107.14	95.06	1.13	144.26	0.74	113.58	0.94
1	72.27	63.79	1.13	102.08	0.71	78.72	0.92
2	43.28	39.61	1.09	65.85	0.66	50.76	0.85
4	25.92	23.63	1.10	39.68	0.65	31.62	0.82
8	15.52	13.70	1.13	22.90	0.68	18.96	0.82
14	10.26	8.71	1.18	14.42	0.71	12.36	0.83
24	6.89	5.60	1.23	9.16	0.75	8.16	0.84

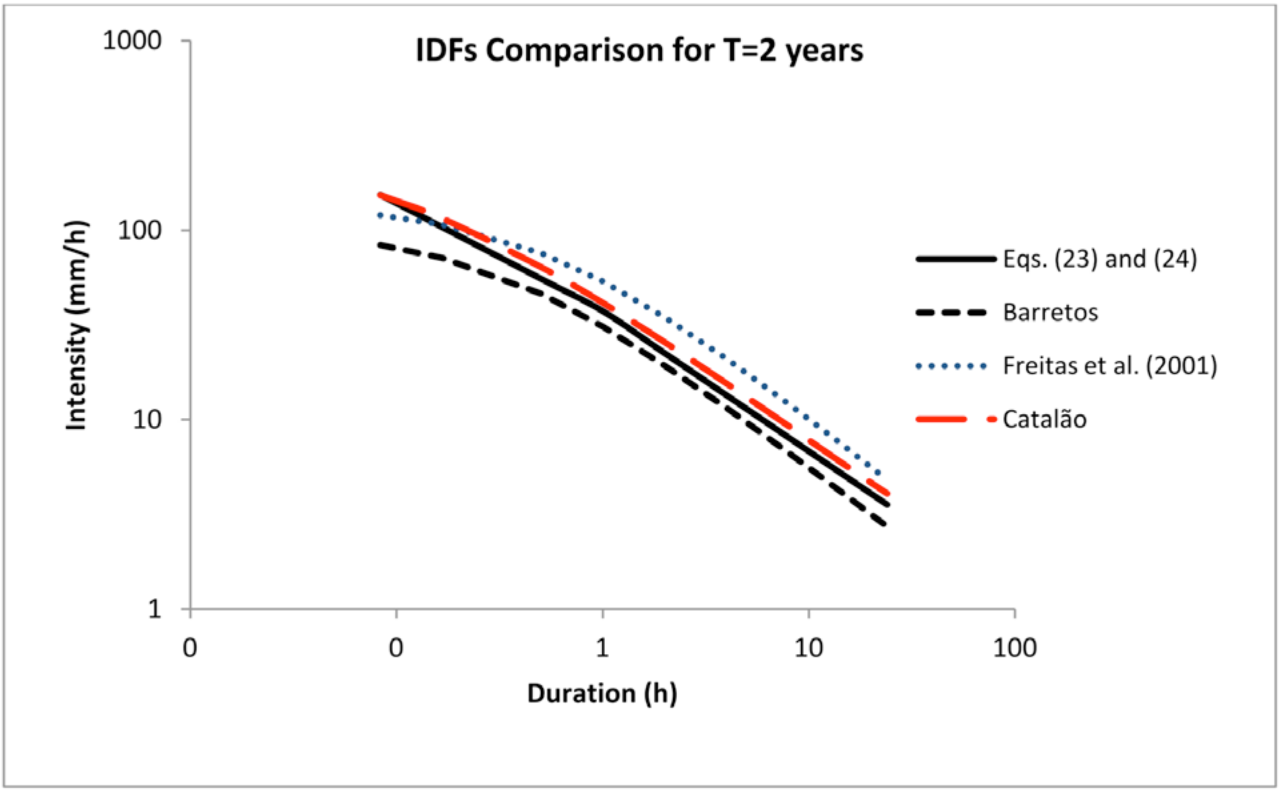

Figure 10 – Comparison among IDF relationships for T=2 years.

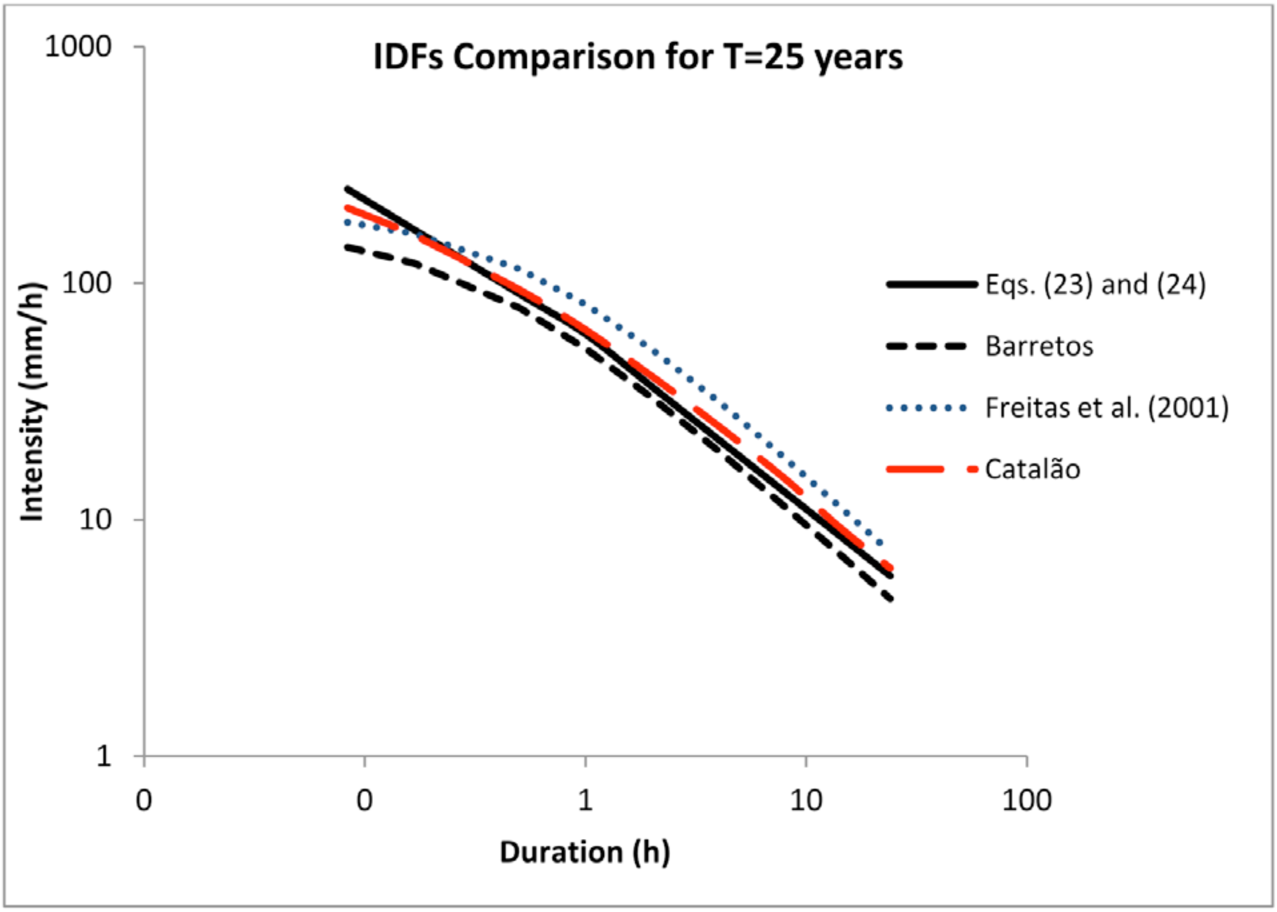

Figure 11 – Comparison among IDF relationships for T=25 years.

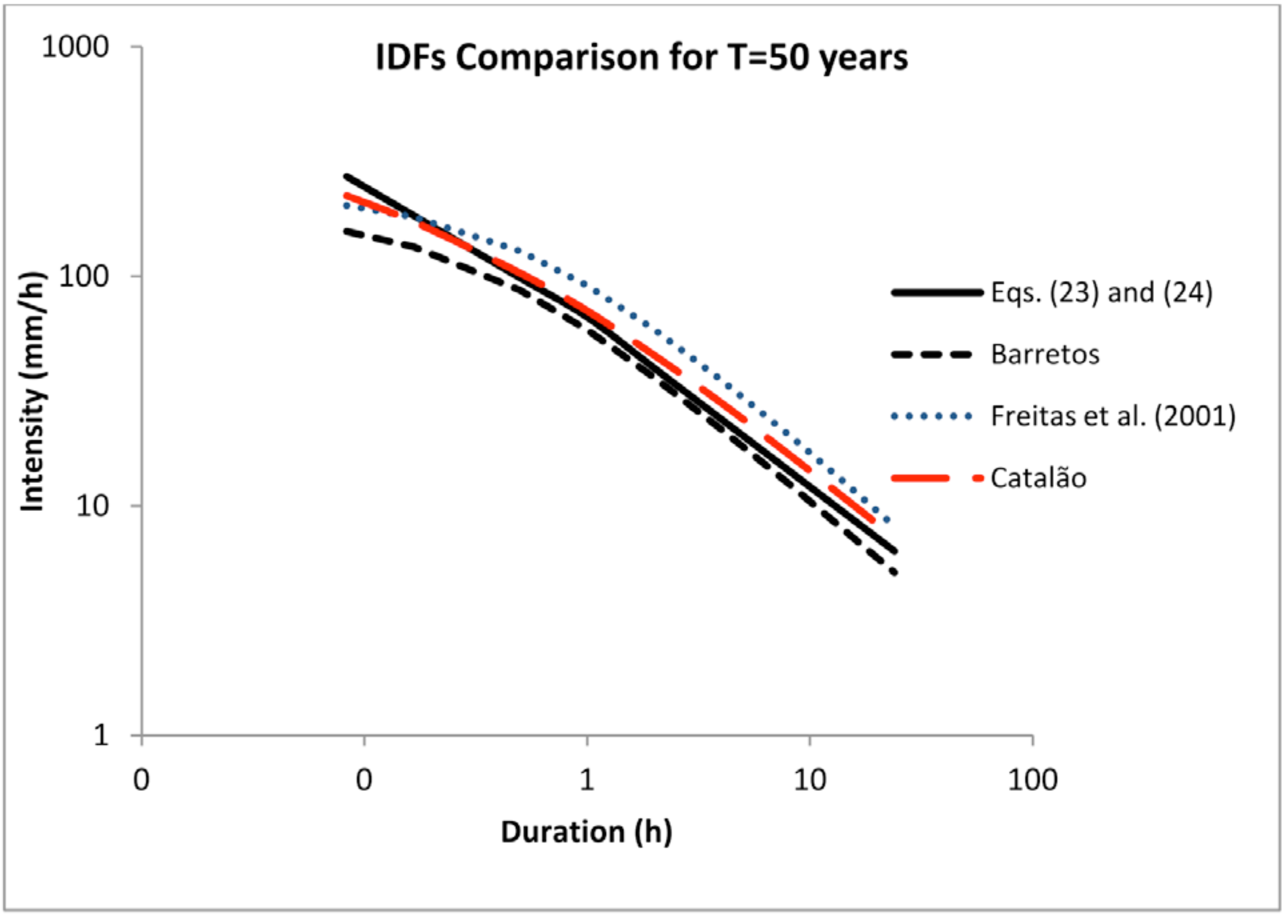

Figure 12 – Comparison among IDF relationships for T=50 years.

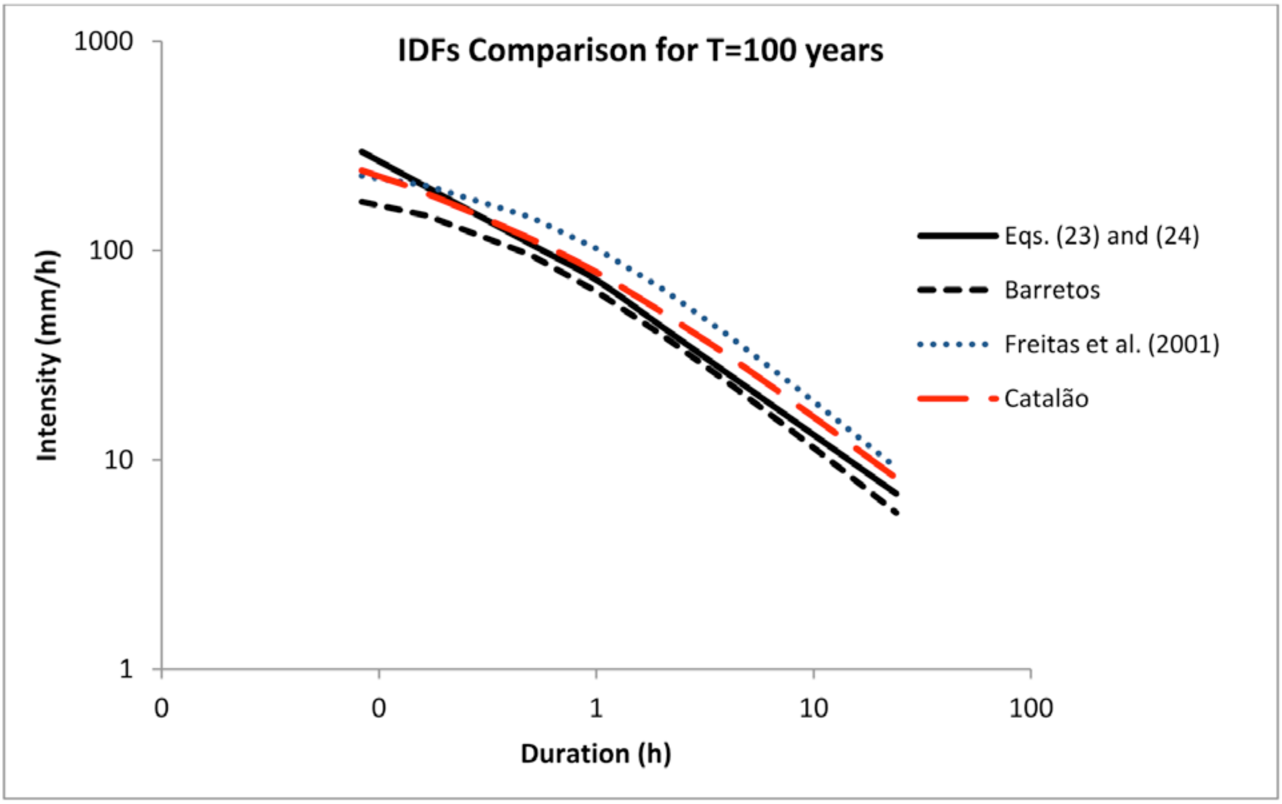

Figure 13 – Comparison among IDF relationships for T=100 years.

7 – ANALYSIS OF THE RESULTS AND CONCLUSIONS

The IDF relationships as estimated in here for the town of Uberaba, formalized by equations (23) and (24), were based on the scale invariance properties, first, for durations from 24 hours to 1 hour, and, then, for durations from 1 hour to 10 minutes. In general, these distinct ranges of durations also indicate significant differences in the prevalent mechanisms of uplifting air masses in the atmosphere, thus often leading to rainfall of the convective type or of the frontal type. Therefore, from this viewpoint, it seems coherent the scale inflection between these ranges of durations, as proposed in this paper.

The deduction of IDF relationships for Uberaba from numerical scale factors estimated for the location of Vespasiano, can be justified by the relatively small variability of these factors, as observed from estimates for other rainfall recording gauging stations situated in the state of Minas Gerais (Naghettini, 2000). In addition, the series of short-duration rainfall rates recorded at the four recording gauging stations considered in here, are consistent with the scale inflection point placed at the duration of 1 hour. These empirical evidences and the arguments from the scale invariance theory provided grounds for deducing IDF estimates from at-site daily rainfall depths recorded at the gauging station of Uberaba. However, since the scale factor η and parameters μ and σ, as in equation (22), may be interpreted as regional climatic characteristics (Menabde, 1999), further in-depth work is needed to elucidate possible covariation between η and climatic and/or relief variables.

The comparison with the results obtained for locations relatively close to Uberaba, like Barretos (north of São Paulo state) and Catalão (south of Goiás state), suggests a certain coherence for the macro-region where Uberaba is placed, known as “Triângulo Mineiro”, bordered on the south by the state of São Paulo and on the north by the state of Goiás. The IDF estimates for Uberaba are relatively close and intermediate between the ones valid for Barretos and Catalão. The IDF estimates resulting from the equation by Freitas et al. (2001), obtained from interpolation among point estimates over a large area, depart significantly from the other results and, thus, seem to be inconsistent.

It is worth to remind, however, this is a preliminary study motivated by the unavailability of at-site sub-daily rainfall data, of adequate time span, at the Uberaba gauging station. In order to check the model of simple scaling for IDF estimation and its results, it is absolutely necessary to install rainfall recording gauging stations in the urban and peri-urban areas of Uberaba, and, once adequate samples of sub-daily rainfall data become available, to proceed with regional frequency analysis of maximum records, by contemporary methods such as the L-Moment procedures described in Hosking and Wallis (1997). Future sub-daily records would eventually provide evidences to further understand the properties of scale invariance, in the context of IDF estimation, and to relate them to local climate and relief characteristics in a more meaningful way.

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