COMPARISON OF MATHEMATICAL ALGORITHMS FOR DETERMINING THE SLOPE ANGLE IN GIS ENVIRONMENT

APLICACIÓN DE ALGORITMOS MATEMÁTICOS EN LA DETERMINACIÓN DE LA INCLINACIÓN DE PENDIENTE EN UN ENTORNO SIG.

José L. García Rodríguez[1] and Martín C. Giménez Suárez[2]

Abstract

Many environmental models depend to a great degree on the accuracy of estimated slope values. A Geographic Information Systems (GIS) can extract slope angles from Digital Elevation Models (DEMs) using slope algorithms. The objective was to verify differences in estimating slope values using nine different mathematical algorithms on 10 m resolution DEMs. Software used were ArcGIS® 9.2 and SEXTANTE®. SEXTANTE® allows selecting the algorithm in order to calculate slope angle values, unlike ArcGIS, which offers only one option.

The results indicated that the 2nd Polynomial Adjustment algorithm of Zevenbergen and Thorne is the most appropriate for the slope angle estimation.

Keywords: ArcGIS, Sextante, slope angle, algorithm, DEM, GIS

Resumen

Muchos de los modelos ambientales dependen en gran medida de la precisión en las estimaciones de pendiente. Un sistema de información geográfica (SIG) puede extraer ángulos de pendiente desde modelos de elevación digital (DEM en inglés) usando los denominados algoritmos de pendiente. En este trabajo se busco verificar diferencias en la estimación del valor de pendiente, calculados a partir de 9 diferentes algoritmos matemáticos sobre DEMs de 10 m de resolución. El software utilizado ha sido los GIS, ArcGIS® 9.2 y SEXTANTE®. Este último permite la posibilidad de poder elegir con que algoritmo poder calcular los valores de pendiente sobre una cuenca, a diferencia de ArcGis® que solo tiene una opción disponible. Los resultados indicaron que el algoritmo de Ajuste de Polinomio de 2º grado de Zevenbergen y Thorne (1987), resultó el más apropiado para la estimación de la inclinación de pendiente.

Palabras Clave: ArcGIS, Sextante, inclinación de pendiente, algoritmo de pendiente, DEM, SIG

INTRODUCTION

The improved accuracy of slope gradient values obtained from Geographic Information Systems (GIS) has a fundamental objective: to contribute to a wide range of environmental models, like erosion models, that have the slope factor as an input.

A GIS can extract slope angles from Digital Elevation Models (or DEMs) using slope algorithms. The effects of slope algorithms over slope angle estimation can vary widely in terms of the accuracy of the calculation.

Objectives

- Objective 1: Confirm differences in estimated slope values, calculated using 9 different mathematical algorithms on DEMs of 10 m resolution.
- Objective 2: Study Root Mean Square Error (RMSE) between each method and field data obtained for three ranges of slopes, 0-5º (9%), 5-20º (9-36%), and >20º (>36%) to verify the slope algorithm that best represents each range.

Material and Methods

The aim of this study was to compare data calculated using GIS and sample points measured in the Arroyo del Lugar basin (Figure 1). To make this possible, a series of slope data was taken in the field, in order to compare them with the data extracted from DEMs (Table 1). An analog clinometer was used in the field to measure the slopes; and a Trimble® GeoExplorer 3 GPS to determine the geographical position. The Topogrid method included in ArcGIS was used to create a DEM from 10 m contour lines.

Software used in this paper were GIS ArcGIS® 9.2 and SEXTANTE® (Olaya, 2006).

One of the GIS used for this study was the recently launched SEXTANTE (Olaya, 2006). It facilitated the modernization, as it offers very significant advantages in terms of the hydrological analysis, in comparison with ArcGIS. One of the most important advantages provided by SEXTANTE is the possibility of selecting the algorithm to calculate slope angle values, as it has several algorithms integrated, unlike ArcGIS,

[1] Professor of Hydrology, Department of Forest Engineering, Hydraulics and Hydrology Laboratory, ETSI Montes, Polytechnic University of Madrid, Spain. josel.garcia@upm.es

[2] Corresponding author. Forestry engineer. Hydraulics and Hydrology Laboratory, Forest Engineering Department, ETSI Montes at the Polytechnic University of Madrid, Ciudad Universitaria s/n (28040), Madrid, Spain. Tel/Fax:+34-913367093, martincgs@ingenieros.com)

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which offers only one option. SEXTANTE is a free software and available in English and Spanish. SEXTANTE is now part of GvSIG package (http://www.gvsig.gva.es/).

Test Area

The basin chosen was the Arroyo del Lugar Basin located in the Municipality of Puebla de Valles, in the northwest section of the Province of Guadalajara, Spain (Figure 1). The total area of Arroyo del Lugar basin is 768.62 ha and total length of the main stream is 7,253 m.

The main characteristic of the basin is the high quantity of gullies with steep slopes.

Methods - Objective 1

Slopes were calculated over a DEM with a resolution of 10 x 10 m, using nine different mathematical algorithms:

a. Neighbourhood Method. Burrough, P. A. and Mcdonell, R.A. Algorithm (1998). Included in ArcGIS.
b. 2nd Degree Polynomial Adjustment. Bauer, Rohdenburg and Bork Algorithm (1985)
c. 2nd Degree Polynomial Adjustment. Heerdegen and Beran Algorithm (1982).
d. 2nd Degree Polynomial Adjustment. Zevenbergen and Thorne Algorithm (1987).
e. 3rd Degree Polynomial Adjustment. Haralick Algorithm (1983)
f. Maximum Slope. Travis Algorithm (1975)
g. Maximum Slope by Triangles. Tarboton Algorithm (1997)
h. Least Squares Fit Plane. Costa-Cabral and Burgess Algorithm (1996)
i. Maximum Downhill Slope. Hickey, Van Remortel and Maichle Algorithm (2004)

The methods named above can be divided into three groups.

The first group consists of methods marked with letters *a* to *e;* i.e. the neighbourhood method and the polynomial methods, which calculate an average value through the central cell, using at least 4 of 8 surrounding cells (Dunn *et al.*, 1998) over a 3 x 3 cells network (Figure 2). This group of algorithms is known as “averaged algorithms”, because they use four or more cells in a network to calculate the slope of the central cell.

The neighbourhood method is the technique incorporated in ArcGIS, to determine slope values (Dunn *et al.,* 1998).

Dunn et al. (1998) mention that the neighbourhood method does not consider the elevation of the central cell. As such, this leads to a certain inaccuracy in slope estimates if the information regarding altitude presents small depressions, peaks, or if the network is centred along a mountain range or valley.

The polynomial adjustment or the quadratic surface method is a partial quadratic equation that can be used to pass through exactly nine elevation points in a three by three grid (Zevenbergen and Thorne, 1987). The slope is the first derivative z (altitude) with regard to the direction of the slope.

This methodology considers only 4 neighbouring cells (z_2, z_4, z_6 and z_8) which are adjacent to the central cell (z_9); consequently its consideration is limited to the local variability surrounding the central cell (Figure 2). In summary, according to Dunn et al. (1998) the same limitations inherent in the neighbourhood method apply to the Polynomial Adjustment methods.

A second group includes the methods labelled from *f* to *h.* These methods are fundamentally associated with flow algorithms, and not with a purely morphometric analysis. They consider the flow moving through a flat surface in the direction of the maximum slope (Suet-Yan Lam, 2004). Due to that, the local morphometry is not defined based on a mathematical function type $z = f(x, y)$, nor are the tools for differential calculus used, as often happens in other cases. As a result, obtaining certain parameters using these methods is not recommendable. Slopes and directions obtained may be valid, although less accurate (Olaya, 2006).

The third group represents algorithms that calculate maximum slope as the direct difference between the central cell and a neighbouring cell. This group, is represented by the Maximum Downstream Slope Algorithm of Van Remortel et al. (2004). Hickey *et al.* (1994) originally created the algorithm for LS factor estimation. LS factor is part of USLE model for hydric erosion calculation. Van Remortel *et al.* (2004) adapted LS factor for RUSLE, i.e., revised USLE model.

This method, unlike the first group, considers the elevation of the central cell (z_9) when estimating slope, and this type of methodology, is known as *non-averaged.* This method proposes that the maximum slope (rise/run relation) between the central cell (z_9) and its eight neighbours (z_1 to z_8) should be used to estimate the slope of the central cell in a 3 x 3 cells network (Dunn *et al.,* 1998).

Methods - Objective 2

For purposes of this study, DEM error at one point is the difference between calculated slope value and its real value. In this case, the accuracy of slope estimations is presented in the form of the Root Mean Square Error (RMSE) statistic expressed as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}\left(S_i^{interpolated} - S_i^{real}\right)^2}{N}} \quad (1)$$

Where, $S_i^{interpolated}$ refers to the i[th] interpolated slope angle value, S_i^{real} refers to the i[th] known or measured slope angle value of a sample point and N is the number of sample points.

In this case, the RMSE was calculated for the slope algorithms studied in Objective 1 (Table 1), for three ranges of slopes, attempting to make calculations for flat, intermediate and steep surfaces.

Dividing the slopes into three ranges allowed us to determine the methodology that best represents the reality of the terrain in each situation, which consecutively shows which model we should choose at the time we undertake a research, according to the type of predominant surface area.

RESULTS

Determine the existence of differences between the slope algorithms groups (field data included).

The analysis revealed that there were no significant differences at the 95% confidence level between all the groups. Statistical values were F=0.690 and p=0.718. Tarboton's Maximum Slope by Triangles Algorithm (maxpend_tri) presented the highest "Maximum" value (Max=29.48) and Van Remortel, Maichle and Hickey Maximum Downstream Slope Algorithm had the greatest variability (Std. Dev.= 8.06).

Kruskal-Wallis analysis confirmed the ANOVA results, indicating no difference between the groups. Statistical values with a 95% confidence level, were χ^2=8.125 and p=0.522.

Determine the existence of relations between each slope algorithm and field data

In order to observe the way in which groups are related, correlation coefficients between pairs of variables were calculated using the Pearson and Spearman correlation.

The best correlation with field data, according both, Pearson and Spearman correlation coefficients, was with Zevenbergen and Thorne 2nd Degree Polynomial Adjustment (Zevenb_AP2) algorithm, with a positive value of 0.671 and 0.721 at 99% confidence level, respectively.

Results of Root Mean Square Error (RMSE) estimation for Slope Algorithms

For smaller slopes than 9%, the polynomial adjustment methods show a tendency for smaller RMSE (Table 2).

RMSE values are similar in the mean slope range (9%-36%) but for slopes bigger than 20 %, RMSE values were disparate.

The row "Total" of Table 2 shows mean RMSE values for each slope algorithm, calculated for the total spectrum of slopes. According to this, the lowest RMSE corresponds to Zevenbergen and Thorne (Zevenb_AP2) algorithm.

Discussion and Conclusions

Since early 1960s, GIS has been used to manage large surfaces of land. A common objective in these management plans has been how to obtain a topographic model. As a result, an accurate estimate of the topography and topographical elements is essential.

The great majority of GIS users, use ArcGIS as the only option. ArcGIS could easily be complemented with other GIS, such as SEXTANTE, which offers calculation variants that are not found in ArcGIS: simply export the DEM made in ArcGIS to SEXTANTE using the *floatgrid* module, apply the slope algorithm, which is appropriate for the study area, reverse this step with the slope raster, and continue working in ArcGIS, if this is the environment preferred by the user.

Tests showed that all algorithms provide similar results of slope angles, but due to the correlation indexes and RMSE values, the recommended algorithm for determining slope angles is the Zevenbergen and Thorne 2nd degree Polynomial Adjustment algorithm (Zevenbergen and Thorne, 1987).

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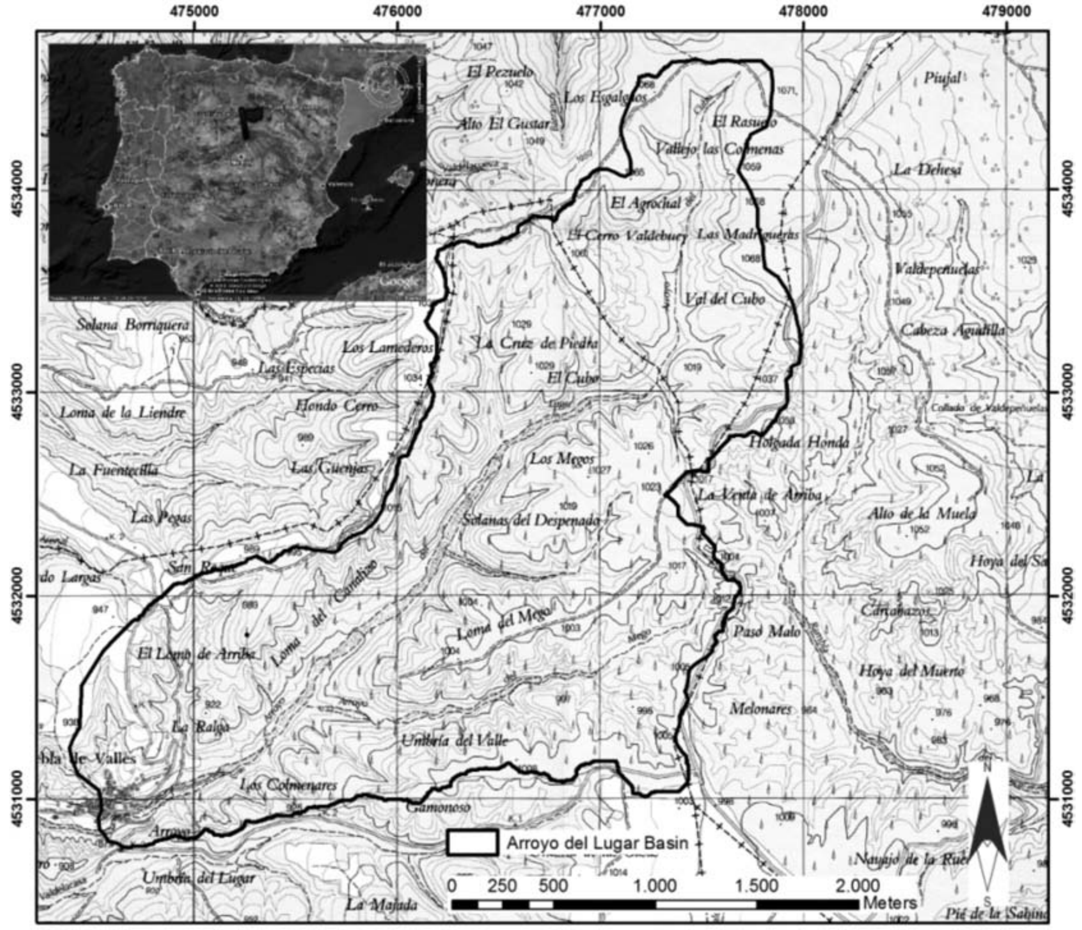

Figure 1. Location of the Arroyo del Lugar basin (Puebla de Valles, Spain)

z1	z2	z3
z8	**z9**	z4
z7	z6	z5

Figura 2. 3 x 3 mask of cells of a raster grid.

Table 1. Slope Values in degrees from nine different algorithms to estimate slope, extracted from nine rasters, with a cell size of 10 m and sample points taken in the field ("Campo10m" column).

Point	Cam-po10m	ArcGIS (S&E)	Bau_ AP2	Zeve_ AP2	Herr_ AP2	Max_pen	Max-pen_tri	Pl_ ajuste	Hara_ AP3	Hick_ mpab
1	22.294	14.864	16.931	15.205	16.931	13.278	13.609	15.128	15.365	14.332
2	7.407	8.514	8.715	8.432	8.715	6.838	10.380	8.118	8.677	9.524
3	16.699	10.179	12.885	10.228	12.885	5.840	8.358	9.613	9.694	5.439
4	14.036	15.444	15.979	14.244	15.979	8.609	14.196	13.564	13.343	11.411
5	1.718	4.424	3.694	3.595	3.694	3.054	2.485	3.473	3.655	2.438
6	12.407	15.283	8.715	12.448	8.715	10.582	9.254	11.605	11.767	14.684
7	30.964	15.640	15.651	14.331	15.651	12.120	9.254	14.263	14.720	15.025
8	1.146	14.897	3.694	7.341	3.694	4.864	5.527	6.857	6.592	11.251
9	30.964	23.494	12.663	24.581	12.663	20.500	20.384	23.822	24.179	19.684
10	20.807	23.926	19.742	23.495	19.742	21.350	21.762	23.086	23.099	26.044
11	20.807	25.910	20.322	27.015	20.322	28.018	22.864	27.072	27.615	24.754
12	19.290	17.582	20.322	19.889	20.322	25.407	21.287	20.002	20.488	28.066
13	14.036	6.675	15.873	7.885	15.873	9.634	17.122	8.042	7.424	6.021
14	6.277	15.133	15.914	12.086	15.914	11.106	19.803	12.218	12.367	13.359
15	4.574	13.120	14.447	19.636	14.447	21.765	15.923	19.767	20.545	15.724
16	11.860	11.161	14.447	11.526	14.447	10.969	19.560	11.623	11.104	10.503
17	1.718	15.041	16.237	12.647	16.237	15.023	19.560	13.009	12.613	20.119
18	6.277	15.275	12.702	12.459	12.702	13.038	18.692	12.827	13.081	14.533
19	6.843	1.025	6.121	4.850	6.121	6.325	10.834	4.925	4.532	1.085
20	2.291	0.754	2.912	0.754	2.912	0.952	4.477	0.717	0.596	0.852
21	1.146	1.459	2.388	3.110	2.388	3.628	5.092	2.919	3.013	1.403
22	1.146	1.922	3.212	2.545	3.212	3.448	5.092	2.647	2.580	2.143
23	9.090	6.766	8.388	10.121	8.388	14.023	11.443	10.476	11.276	11.157
24	1.146	1.348	3.932	2.048	3.932	2.495	6.757	2.008	1.924	1.107
25	4.004	5.618	9.704	6.903	9.704	7.595	14.438	6.804	6.075	5.551
26	5.143	3.825	10.018	6.232	10.018	8.003	15.321	6.073	5.983	5.046
27	7.407	14.015	10.581	11.878	10.581	11.535	10.988	11.618	11.716	12.022
28	7.407	7.455	10.581	8.412	10.581	7.333	20.832	8.220	7.853	5.573
29	6.843	7.789	10.492	9.778	10.492	13.099	20.832	9.800	9.897	10.279
30	7.970	12.828	12.662	15.093	12.662	14.105	20.737	14.472	14.651	11.234
31	14.574	16.128	13.560	15.612	13.560	20.503	24.385	15.609	15.795	23.584
32	11.310	24.858	21.889	27.826	21.889	29.487	24.385	27.317	27.633	26.406

Note: Campo10m: Field data; ArcGIS(S&E): Burrough, P. A. and Mcdonell, R.A. Alg. (1998); Bau:AP2: Bauer, Rohdenburg and Bork Alg. (1985); Herr_AP2: Heerdegen and Beran Alg. (1982); Max_pen: Travis Alg. (1975); Maxpen_tri: Tarboton Alg. (1997); Pl_ajuste: Costa-Cabral and Burgess Alg. (1996); Zeve_AP2: Zevenbergen and Thorne Alg. (1987); Hara_ AP3: Haralick Alg. (1983); Hick_mpab: Van Remortel, Maichle and Hickey Alg. (2004).

Table 2. Root Mean Square Error (RMSE) values, with regard to field data, for each one of the 9 slope algorithms, extracted from 9 rasters with a cell size of 10 m. Smallest RMSE is indicated in shady.

Slope Ranges	ArcGIS (S&E)	Bau_AP2	Zeve_AP2	Max_pen	Maxpen_tri	Pl_ajuste	Herr_AP2	Hara_AP3	Hick_mpab
0-5° (9%)	7.09	6.37	6.71	6.37	7.57	8.45	6.75	6.84	7.97
5-20° (9%-36%)	5.61	4.55	5.34	4.55	6.54	8.95	5.27	5.44	6.66
>20° (>36%)	8.74	10.95	9.09	10.95	10.94	11.53	9.23	9.05	9.88
Total	6.61	6.46	6.45	6.46	7.67	9.27	6.46	6.52	7.62